

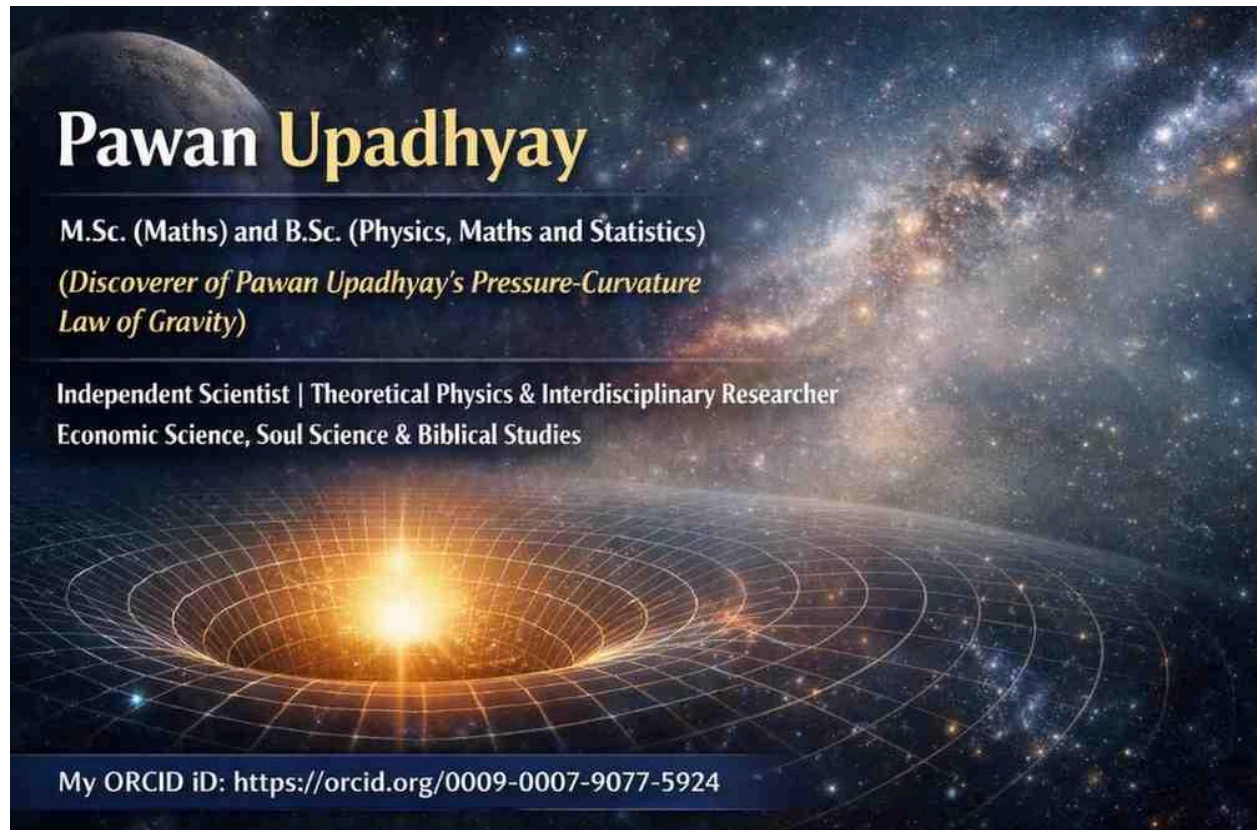
Mathematical Relationship Between the Gravitational Pressure Field, Pressure Gradient Field, and Gravitational Field in Pawan Upadhyay's Pressure Curvature Law of Gravity

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Abstract

This paper presents the mathematical relationship between the Gravitational Pressure Field, the Pressure Gradient Field, and the Gravitational Field within the framework of Pawan Upadhyay's Pressure Curvature Law of Gravity (PPC Law of Gravity). The theory introduces the gravitational pressure field as the fundamental physical quantity governing gravity. Spatial variations of this pressure generate a pressure gradient field, which produces gravitational force density in the weak-field regime and four-field force density in the relativistic strong-field regime. Consequently, both the classical gravitational field and relativistic four-acceleration emerge from a common pressure-based description of gravity.

1. Introduction

The PPC Law of Gravity proposes that gravity originates from gravitational pressure rather than from a fundamental attractive force. The theory introduces a scalar Gravitational Pressure Field whose spatial and temporal variations govern gravitational motion.

Unlike Newtonian gravity, where the gravitational field is regarded as fundamental, the PPC framework identifies the gravitational pressure field as the primary field, while the gravitational field is a derived quantity.

2. Gravitational Pressure Field

The gravitational pressure field is defined as

$$P_g = \omega E_d,$$

where

- P_g is the gravitational pressure,
- E_d is the energy density,
- ω is the equation-of-state parameter.

This scalar field represents the gravitational pressure distribution throughout spacetime.

3. Relativistic Momentum Conservation

Projecting the conservation equation perpendicular to the four-velocity gives the momentum conservation equation.

The relativistic acceleration equation becomes:

$$(E_d + P_g)a^\nu + \nabla^\nu P_g = 0$$

where:

$$a^\nu = u^\mu \nabla_\mu u^\nu$$

is the four-acceleration.

This equation is central to the PPC theory.

3. Pressure Gradient Field

The pressure gradient field is obtained through the gradient of the gravitational pressure field,

$$\nabla P_g$$

in the weak-field limit, and

$$\nabla^\nu P_g$$

in the relativistic strong-field limit.

The pressure gradient field represents the spatial and spacetime variation of gravitational pressure.

If

$$\nabla P_g = 0,$$

or

$$\nabla^\nu P_g = 0,$$

the gravitational pressure is uniform, and no gravitational acceleration is produced.

4. Weak-Field Limit

In the weak-field regime, the gravitational field force density is defined as

$$\mathbf{F} = -\nabla P_g.$$

Since

$$\mathbf{F} = \rho \mathbf{g},$$

where ρ is the mass density,

the gravitational field becomes

$$\mathbf{g} = -\frac{\nabla P_g}{\rho}.$$

Thus, the classical gravitational field is directly generated by the pressure gradient field.

5. Strong-Field (Relativistic) Limit

Projecting the relativistic conservation equation perpendicular to the four-velocity yields

$$(E_d + P_g)a^\nu + \nabla^\nu P_g = 0,$$

where

$$a^\nu = u^\mu \nabla_\mu u^\nu$$

is the four-acceleration.

The relativistic equation of motion becomes

$$a^\nu = -\frac{\nabla^\nu P_g}{E_d + P_g}.$$

The relativistic four-field force density is defined as

$$F^\nu = -\nabla^\nu P_g.$$

Therefore,

$$F^\nu = (E_d + P_g)a^\nu.$$

This equation represents the relativistic counterpart of the weak-field relation

$$\mathbf{F} = \rho \mathbf{g}.$$

6. Mathematical Connection Between the Fields

The PPC Law of Gravity establishes a unified mathematical structure.

Weak-field hierarchy

$$E_d \longrightarrow P_g \longrightarrow \nabla P_g \longrightarrow \mathbf{F} \longrightarrow \mathbf{g}$$

where

$$\mathbf{F} = -\nabla P_g,$$

and

$$\mathbf{g} = \frac{\mathbf{F}}{\rho}.$$

$$P_g \xrightarrow{\nabla} -\nabla P_g = \mathbf{F} \xrightarrow{/\rho} \mathbf{g}$$

or in words:

- **Gravitational Pressure Field** P_g is the fundamental scalar field.
- **Pressure Gradient Field** ∇P_g is the spatial variation of the pressure field.
- **Gravitational Field Force Density** is

$$\mathbf{F} = -\nabla P_g.$$

$$\mathbf{g} = \frac{\mathbf{F}}{\rho} = -\frac{\nabla P_g}{\rho}.$$

This sequence shows that the gravitational field is not fundamental but is derived from the pressure gradient generated by the gravitational pressure field.

Strong-field hierarchy

$$E_d \longrightarrow P_g \longrightarrow \nabla^\nu P_g \longrightarrow F^\nu \longrightarrow a^\nu$$

where

$$F^\nu = -\nabla^\nu P_g,$$

and

$$a^\nu = \frac{F^\nu}{E_d + P_g}.$$

Thus, the gravitational pressure field generates the pressure gradient field, which produces the gravitational force density in the weak-field regime and the relativistic four-field force density in the strong-field regime.

7. Physical Interpretation

Within the PPC Law of Gravity,

- The **Gravitational Pressure Field** is the fundamental scalar field.
- The **Pressure Gradient Field** describes the variation of gravitational pressure.
- In the **weak-field limit**, the pressure gradient produces the gravitational field force density, leading to the classical gravitational field.
- In the **strong-field limit**, the spacetime pressure gradient produces the relativistic four-field force density, leading to four-acceleration.

Accordingly, the classical gravitational field and relativistic four-acceleration are two limiting manifestations of the same underlying gravitational pressure field.

Weak-field limit

Strong-field limit

Gravitational field: $\mathbf{g}$

Four-acceleration: a^ν

Field force density: $\mathbf{F}$

Four-field force density
 F^ν

$$\mathbf{F} = -\nabla P_g$$

$$F^\nu = -\nabla^\nu P_g$$

$$\mathbf{F} = \rho \mathbf{g}$$

$$F^\nu = (E_d + P_g) a^\nu$$

This presentation clearly separates the **classical (weak-field)** description from the **relativistic (strong-field)** description while showing that both arise from the same underlying gravitational pressure field.

Note: Local Covariant Gravitational Pressure Field in the PPC Law of Gravity

Within the framework of **Pawan Upadhyay's Pressure Curvature Law of Gravity (PPC Law of Gravity)**, the gravitational pressure field is defined locally at every spacetime point. In curved spacetime, the PPC equation of state preserves its **local covariant form**, ensuring that the theory remains valid in arbitrary coordinate systems.

The local gravitational pressure field is given by

$$P_g(x^\mu) = \omega(x^\mu) E_d(x^\mu), \quad x^\mu \in (M, g_{\mu\nu}).$$

where

- $P_g(x^\mu)$ is the **local gravitational pressure field**,
- $E_d(x^\mu)$ is the **local total energy density field**,
- $\omega(x^\mu)$ is the **local dimensionless equation-of-state parameter**,
- $x^\mu = (ct, x, y, z)$ denotes the spacetime coordinates,
- $(M, g_{\mu\nu})$ represents the curved spacetime manifold equipped with the metric tensor $g_{\mu\nu}$.

The dependence on x^μ indicates that the gravitational pressure field, energy density, and equation-of-state parameter may vary from one spacetime point to another.

Consequently, the gravitational pressure field forms a smooth scalar field over spacetime.

Since gravity in the PPC theory is driven by variations in gravitational pressure, the local gravitational pressure field serves as the fundamental scalar quantity from which the pressure gradient field, gravitational field force density, and relativistic four-field force density are derived. This local covariant formulation ensures that the PPC Law of Gravity is compatible with the geometric description of curved spacetime while preserving the same physical equation of state at every spacetime point.

Key implication: The equation

$$P_g(x^\mu) = \omega(x^\mu)E_d(x^\mu)$$

is not merely a local constitutive relation; it establishes the fundamental gravitational pressure field of the PPC theory, from which both the weak-field and strong-field gravitational dynamics are constructed.

For most physics papers, however, it is common and acceptable to write simply

$$P_g(x^\mu) = \omega(x^\mu)E_d(x^\mu), \quad x^\mu \in M,$$

where $x^\mu = (ct, x, y, z)$ denotes the spacetime coordinates of a point in the manifold.

Note: Relationship Between the Gravitational Potential and the Gravitational Pressure Field in the Weak-Field Limit

Within the weak-field limit of the **Pawan Upadhyay Pressure Curvature (PPC) Law of Gravity**, the classical gravitational field is directly related to the gravitational pressure field.

The Newtonian gravitational field is defined by

$$\mathbf{g} = -\nabla\phi,$$

where ϕ is the gravitational potential.

In the PPC framework, the gravitational field force density is defined as

$$\mathbf{F} = -\nabla P_g,$$

where P_g is the gravitational pressure field.

Since the field force density is related to the gravitational field by

$$\mathbf{F} = \rho \mathbf{g},$$

where ρ is the mass density, substitution yields

$$\rho \mathbf{g} = -\nabla P_g.$$

Therefore,

$$\mathbf{g} = -\frac{\nabla P_g}{\rho}.$$

Using the Newtonian definition of the gravitational field,

$$\mathbf{g} = -\nabla\phi,$$

gives

$$-\nabla\phi = -\frac{\nabla P_g}{\rho}.$$

Hence,

$$\nabla\phi = \frac{\nabla P_g}{\rho}.$$

Integrating both sides leads to

$$\phi = \frac{P_g}{\rho} + C,$$

where C is an arbitrary integration constant determined by the choice of the zero of gravitational potential. Choosing the reference potential such that $C = 0$, the relation becomes

$$\phi = \frac{P_g}{\rho}.$$

This equation establishes a direct relationship between the **gravitational potential** and the **gravitational pressure field** within the weak-field limit of the PPC Law of Gravity. It shows that the classical gravitational potential may be interpreted as the gravitational pressure per unit mass density, thereby connecting the Newtonian potential formulation with the pressure-based description of gravity proposed in the PPC framework.

Key Result

$$P_g \longrightarrow \phi = \frac{P_g}{\rho} \longrightarrow \mathbf{g} = -\nabla\phi$$

This sequence demonstrates that, in the weak-field limit, the gravitational pressure field determines the gravitational potential, whose spatial gradient produces the observable gravitational field.